③ Eg: If $A = \ell^2/4\pi = \pi r^2$, $\ell = 2\pi r$

And $(xy' - \bar{y}x')^2 = (x^2+\bar{y}^2)(x'^2+y'^2)$

$\Leftrightarrow \cancel{x^2y'^2} + \cancel{\bar{y}^2x'^2} - 2xx'\bar{y}y' = x^2x'^2 + \bar{y}^2y'^2 + \cancel{x^2y'^2} + \cancel{x'^2\bar{y}^2}$

$\Leftrightarrow x^2x'^2 + \bar{y}^2y'^2 - 2xx'\bar{y}y' = 0 \Leftrightarrow (xx'+\bar{y}y')^2 = 0$

So $\underbrace{\frac{x}{y'} = -\frac{\bar{y}}{x'}}_{\text{b/c } (x,\bar{y}) \in \text{Circle... this describes the curvature.}} = \pm r$ so $x = \pm r y' \Leftrightarrow x^2+y^2 = \frac{1}{r^2}$?

∎

4-Vertex vx: when $\frac{dk_s}{dt} = 0$ or $\dot{k}_s = 0$.

Recall: $\dot{n}_s = -k_s \dot{t}$

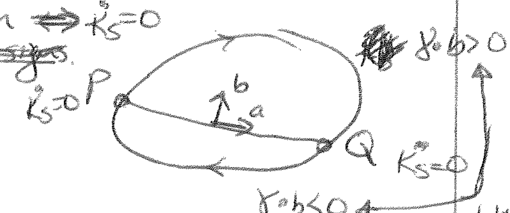
Thm: Every SCC in $\mathbb{R}^2$ has at least 4 vxs.

Pf: Assume γ unit speed, period ℓ.

Then $\int_0^\ell \dot{k}_s \gamma \, dt = \cancel{k_s \gamma}\Big|_0^\ell - \int_0^\ell k_s \dot{\gamma}\, dt = -\int_0^\ell k_s t \, dt = \int_0^\ell \dot{n}_s \, dt = n_s(\ell) - n_s(0) = 0.$

$k_s \neq 0$ (else trivial), so let P, Q be s.t. k_s is max/min $\Rightarrow \dot{k}_s = 0$ ~~has opp signs~~.

Define $\vec{a}, \vec{b}$ as in the picture.



Then, $\int_0^\ell \dot{k}_s (\vec{\gamma} \cdot \vec{b})\, dt = 0$

implies $\dot{k}_s$ and $\vec{\gamma} \cdot \vec{b}$ must have "balancing" signs... need @least 4 vertices (regions) to make this happen! ∎

$\dot{k}_s$ must have same sign on these two segments somewhere $\Rightarrow$ another vx.