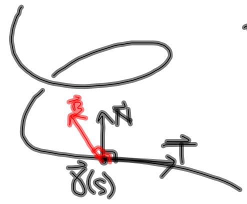

$\gamma(s)$ = unit-spd curve
$\vec T$ = unit tangent
$(\vec T \cdot \dot{\vec T} = 0)$

$\dot{\vec T} = \kappa \vec N$ "curvature"

$\vec B = \vec T \times \vec N$

$\{\vec T, \vec N, \vec B\}$ orthonormal "Frenet Frame"

$\dot{\vec B} = -\tau \vec N$ "torsion"

$\dot{\vec B} \perp \vec B$

$\vec N = \vec B \times \vec T$

$\dot{\vec N} = \dot{\vec B} \times \vec T + \vec B \times \dot{\vec T}$

$= \tau \vec B - \kappa \vec N$

$\dot{\vec B} = \underbrace{\cancel{\dot{\vec T} \times \vec N}}_{0} + \vec T \times \dot{\vec N} \Rightarrow \dot{\vec B} \perp \vec T$

Frenet-Serret Diff Eqns
$\dot{\vec T}, \dot{\vec N}, \dot{\vec B}$ in terms of $\vec T, \vec N, \vec B$

Jan 21-10:04 AM

$$\begin{cases} \dot{\vec T} = \kappa \vec N \\ \dot{\vec N} = \tau \vec B - \kappa \vec T \\ \dot{\vec B} = -\tau \vec N \end{cases} \rightsquigarrow \begin{pmatrix} \dot T \\ \dot N \\ \dot B \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}}_{\text{Skew Symmetric}} \begin{pmatrix} T \\ N \\ B \end{pmatrix}$$

Thm 2.1: Planar curves det'd "uniquely" (rigid motion) by signed curvature.

Thm 2.3: Space curves det'd "uniquely" (rigid motion) by curvature, torsion

- Uniqueness: same rotation works everywhere
- Existence: F-S diff eqs have unique soln.
  Given $\vec T$, $\gamma = \int_{s_0}^{s} \vec T(u)\, du$

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What do curves look like?

- if $\kappa = 0$, planar: line (Prop 1.1) $(\dot{\vec T} = 0)$
- if $\kappa = C$, planar: circle (Def 2.1)

- if $\kappa = 0$, space: line
- if $\tau = 0$: in some plane ($\vec B$ is normal vec. of plane)
- if $\tau = 0$, $\kappa = C$: circle
- if $\tau = C$: ??

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