Ch. 2.3

Frenet-Serret Eqns:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

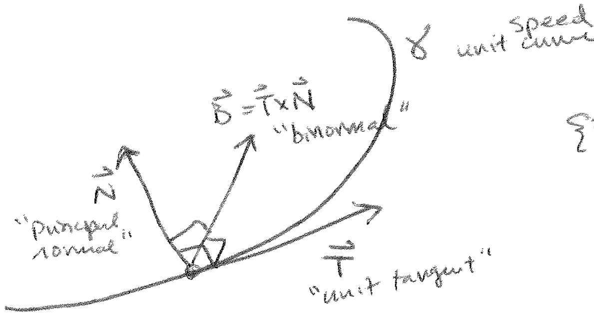
$\vec{\gamma}: \mathbb{R} \to \mathbb{R}^3$ a unit-speed curve.

Then, $\begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix}' = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix}$, i.e.
$$\dot{\vec{T}} = \kappa \vec{N}$$
$$\dot{\vec{N}} = -\kappa \vec{T} + \tau \vec{B}$$
$$\dot{\vec{B}} = -\tau \vec{N}$$

skew-symmetry ensures that the set of vectors remains orthonormal.

i.e. $\begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix}_2 = \begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix}_1 + \delta \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}\begin{pmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{pmatrix} = \begin{pmatrix} \vec{T} + \kappa\delta\vec{N} \\ \vec{N} - \delta\kappa\vec{T} + \delta\tau\vec{B} \\ \vec{B} - \delta\tau\vec{N} \end{pmatrix}$

As $\delta \to 0$, the approximate values become orthonormal.
Algebraically: SO(3) has Lie gp. $\Leftrightarrow$ skew-symm. mxs.



$\{\vec{T}, \vec{N}, \vec{B}\}$ is the "Frenet Frame"
an orthonormal basis @ every pt. of curve
independent of parametrization

$\|\dot{\vec{T}}(s)\| = \kappa(s)$, so let $\vec{N}(s) = \frac{1}{\kappa(s)}\dot{\vec{T}}(s)$ = unit vector in direction of $\dot{\vec{T}}$.

(Recall: since $\vec{T}$ is unit vector, $\|\vec{T}\|^2 = \vec{T}\cdot\vec{T} = 1$
so $\frac{d}{ds}(\vec{T}\cdot\vec{T}) = 2\vec{T}\cdot\dot{\vec{T}} = 0$
so $\vec{T}, \dot{\vec{T}}$ are orthogonal)

$\dot{\vec{B}}(s) = -\tau(s)\vec{N}(s)$ since $\dot{\vec{B}}, \vec{N}$ are parallel.
$\tau(s)$ is the "torsion"

$\kappa = 0 \Leftrightarrow$ line
$\kappa = C_1 \Leftrightarrow$ circle (if planar)
$\kappa = C_1, \tau = 0 \Leftrightarrow$ circle

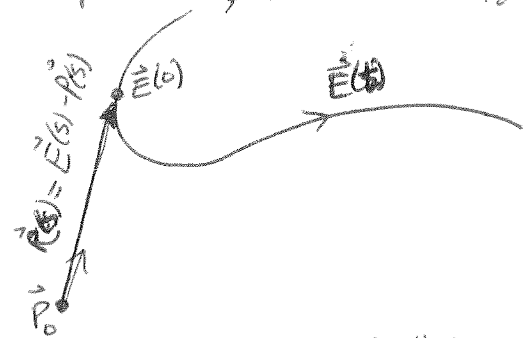
$\tau = 0 \Leftrightarrow$ planar curve
$\tau = C_1 \Leftrightarrow$???
$\tau = C_1, \kappa = D \Leftrightarrow$???

<u>Thm:</u> Unit-speed curves in $\mathbb{R}^2$ are uniquely det'd (up to rigid motion) by (signed) curvature κ_s.
(and any curvature (smooth) is possible)

<u>Thm:</u> Unit speed curves in $\mathbb{R}^3$ are uniquely det'd (up to rigid motion) by curvature $\kappa > 0$ and torsion τ
(and any smooth fns for κ, τ are possible).

Pursuit Curves:

Let $\vec{E}(t)$ be a ~~unit speed~~ curve, take $\vec{P}_0$



Then, heading directly toward $\iff \vec{P}_0' \parallel \vec{R} \iff \vec{P}_0' = S_p(t)\dfrac{\vec{R}}{\|\vec{R}\|}$

~~@ max speed~~