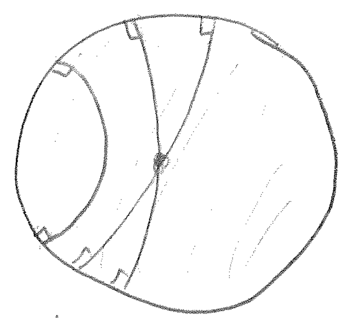
13-Jan-09

Euclidean	Spherical	Hyperbolic

$A = \pi r^2$		
Zero curvature	Positive curvature	Negative curvature
Parallel lines Never int.	Parallel "lines" Intersect	∞ # of Non-int. parallel "lines"

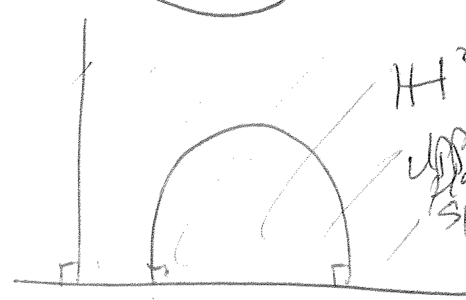
("Line" = geodesic = shortest dist. btwn 2 pts)

Symmetries:
$$z \mapsto \frac{az+b}{cz+d}$$
(Möbius or fractional linear transformations)



$\mathbb{D}^2$

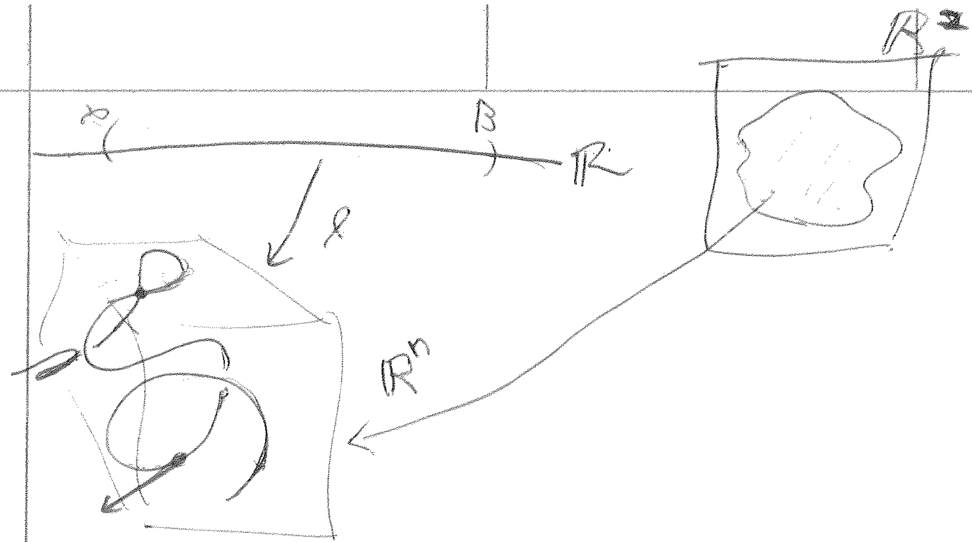
Poincaré Disk Model

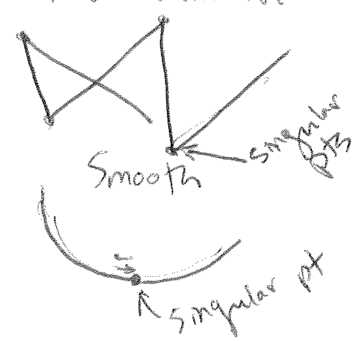


$\mathbb{H}^2$

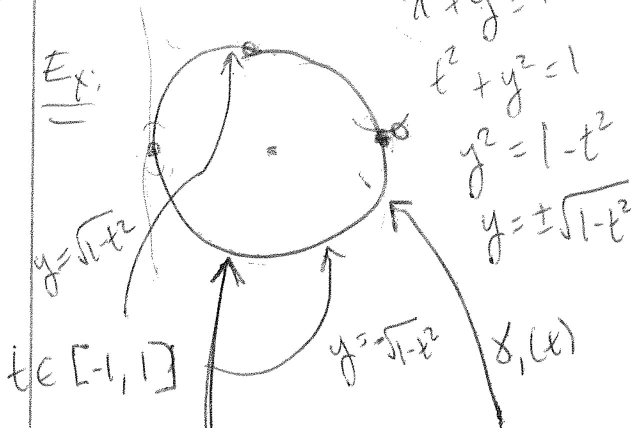
Upper Half Space

TOPS 35500





Ex:



$x^2 + y^2 = 1$

$t^2 + y^2 = 1$

$y^2 = 1 - t^2$

$y = \pm\sqrt{1-t^2}$

$\langle x(t), y(t)\rangle = \langle \cos t, \sin t\rangle$

$\gamma_2(t) = \langle \cos t^2, \sin t^2\rangle \quad \phi(t) = t^2$

$\gamma_2(t) = \gamma_1(\phi(t)) \qquad \dot\gamma_2 = \dot\gamma_1(\phi(t))\,\dot\phi(t)$

$$\begin{array}{ccc} \mathbb{R} & \xrightarrow{\gamma_1} & \mathbb{R}^2 \\ \phi\downarrow & \nearrow_{\gamma_2} & \\ \mathbb{R} & & \end{array}$$

$\gamma_1 = \gamma_2 \circ \phi$

$\gamma_1(t) = \gamma_2(\phi(t))$

$L = \int_a^b \|\dot\gamma\|\,dt$ If $\|\dot\gamma\| = 1$, then $L = \int_a^b dt = b - a.$

(curve is parametrized by Arc Length)

Inv. Fn. Thm: inverse exists when no zero derivative